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AN ANALYTICAL APPROACH TO DETERMINING THE DYNAMIC CHARACTERISTICS OF A CYLINDRICAL SHELL WITH CLOSING CRACKS

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The paper is devoted to developing mathematical models of the elastic oscillations of a cylindrical shell with surface closing cracks. The respective forms of shell vibrations have been chosen to represent various types of damage of the shell. In the case of dispersed and single-surface damage, the transverse shell vibrations are simulated. The cycle of vibrations is assumed to be subdivided into two parts, in one of them the damaged surface fibers are compressed so closing the cracks and negating their influence. For the second part, the cracks are open, so their influence is taken into account. The problem is solved in a piecewise linear with different frequencies and amplitudes at each vibrations cycle interval. The vibration parameters are calculated by means of Relay's energy conservation method and are represented by analytical expressions, the system being assumed to be conservative. The functions determining the vibration process are decomposed by a Fourier analysis using the averaged frequency, the coefficients of the resulting series being obtained as analytical expressions. Vibrodiagnostic functions, which enable the geometrical parameters of the cracks to be determined depending on the geometry of the shell and type of damage, have been plotted.

1. INTRODUCTION

The experimental and theoretical methods of research of bending vibrations of construction elements (beams) with a closing crack have been investigated in the papers [1–8]. It has been mentioned that researches [8, 9] are devoted to the above said problem. The vibrations of simply supported homogeneous beams with a crack, represented as an equivalent spring with linear parameters connecting the two beam segments without damages, are considered. The coefficients of stress intensity factors for cylinder shells with the open axial, circular and arbitrarily oriented cracks of finite length are considered in the handbook [10]

Papers [11–15] are devoted to the study of cylinder shells vibrations. The comparison analysis of the results of determining the influence zone of the crack when considering the problem of vibration of a supported beam has been performed in paper [11]. A problem of shell vibrations with dispersed surface cracks has been considered in research [12]. The ratio of amplitude of the second harmonic appearing during vibrations to the amplitude of the first harmonic has been found. The ratio of the amplitude of shell vibrations with a transverse crack to that of shell vibrations without crack for one of the possible variants of the cracks influence zone on the parameters of the shell vibrations has been found. The shell vibrations with a longitudinal crack have been discussed in studies [14, 15]. The experimental researches of the diagnostic parameters for beam vibrations have been stated in references [16, 17].

Of note are the papers of Tsyfansky and Beresnevich [18] and Staszewski and Worden [19] in which the damage detection in construction elements has been considered.

The purpose of this paper is the determination of the parameters of bending vibrations (frequencies, amplitudes) of circular cylindrical shells without cracks, having transverse isolated cracks and dispersed closing cracks. The results obtained have been used for the diagnosis of cracks. The parameters which control diagnostic functions have been proposed.

2. THE EQUATIONS AND SUPPOSITIONS

Equations of vibrations of the undamaged circular cylindrical shell are, from reference [20],

$$\frac{\partial^2 u}{\partial x^2}+\frac{1-\mu}{2}\frac{\partial^2 u}{\partial y^2}+\frac{1+\mu}{2}\frac{\partial^2 v}{\partial x\partial y}-\frac{\mu}{a}\frac{\partial w}{\partial x}+\frac{1-\mu^2}{Eh}p_x-\rho\frac{1-\mu^2}{E}\frac{\partial^2 u}{\partial t^2}=0,$$

$$\frac{1+\mu}{2}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{1-\mu}{2}\frac{\partial^2 v}{\partial x^2}-\frac{1}{a}\frac{\partial w}{\partial y}+\frac{1-\mu^2}{Eh}p_y-\rho\frac{1-\mu^2}{E}\frac{\partial^2 v}{\partial t^2}=0,$$

$$\frac{h^2}{12}\nabla^4 w-\frac{\mu}{a}\frac{\partial u}{\partial x}-\frac{1}{a}\frac{\partial v}{\partial y}+\frac{w}{a^2}-\frac{1-\mu^2}{Eh}q+\rho\frac{1-\mu^2}{E}\frac{\partial^2 w}{\partial t^2}=0, \tag{1}$$

where the x-axis is directed along the symmetry axis of the median shell surface, the y-axis in the circular direction, the z-axis along the interior normal of the median shell surface, u is the component of transition along the x-axis, v is the component of transition along the y-axis, w the component of transition along the z-axis, μ is the Poisson coefficient, a is the radius of median shell surface, E is Young's modulus, h is the shell thickness, ρ is the density, p_x is the load intensity along the x-axis, p_y the load intensity along the y-axis, q the load intensity along the z-axis, t is the time of vibrations, $\nabla^4=\nabla^2\nabla^2=\partial^4/\partial x^4+2\,\partial^4/\partial x^2\partial y^2+\partial^4/\partial y^4$.

When studying natural frequencies and forms of shell vibration in equations (1) $p_x=0$, $p_y=0$, $q=0$ are assumed. The dimensionless variables $\xi=x/a$, $\varphi=y/a$ are introduced.

In case of axisymmetric vibrations the solution of equations (1) are:

$$u=A_m\cos\lambda_m\xi\cos n\varphi\cos\omega t,\quad v=B_m\sin\lambda_m\xi\sin n\varphi\cos\omega t,\quad w=C_m\sin\lambda_m\xi\cos n\varphi\cos\omega t, \tag{2}$$

where n is the number of waves in a circular direction, ω the natural frequency of vibrations, λ_m the frequency parameter, m the number of half waves in the longitudinal direction (Figure 1).

It is assumed, that two frequencies will appear in a full period of the vibration cycle, ω_0 which is appropriate to the case of closed cracks and ω_1 in case of the opened cracks.

The full period of shell vibrations may be written as $T=\pi/\omega_0+\pi/\omega_1=\pi(\omega_1+\omega_0)/\omega_1\omega_0$. The averaged frequency of vibrations is $\bar{\omega}=2\pi/T=2\omega_1\omega_0/(\omega_1+\omega_0)$. The function describing the vibrations of the shell with a crack differ in the parts of shell vibrations with the opened and closed cracks. In case of the transverse vibrations it can be

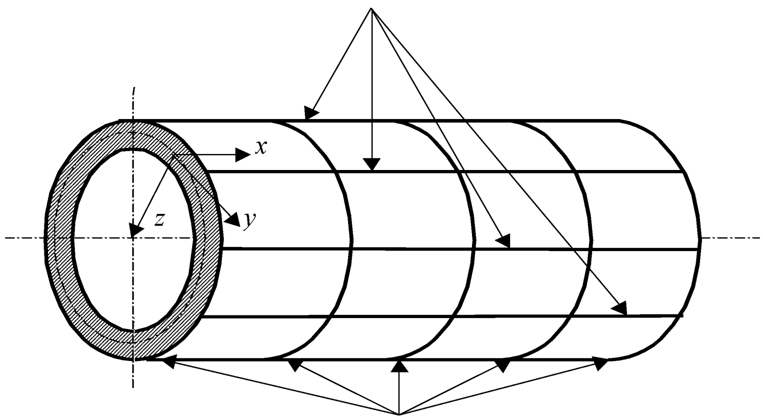

Figure 1. Model of the shell.

written, for example, as

$$w = \begin{cases} A_0 \sin \dfrac{\pi x}{l} \cos \omega_0 t, & 0 \leqslant t \leqslant \dfrac{\pi}{2\omega_0} \\ A_1 \sin \dfrac{\pi x}{l} \cos \omega_1 \left(t + \dfrac{\pi}{2\omega_1} - \dfrac{\pi}{2\omega_0} \right), & \dfrac{\pi}{2\omega_0} < t \leqslant \dfrac{\pi}{2\omega_0} + \dfrac{\pi}{\omega_1} \\ A_0 \sin \dfrac{\pi x}{l} \cos \omega_0 \left(t + \dfrac{\pi}{\omega_0} - \dfrac{\pi}{\omega_1} \right), & \dfrac{\pi}{2\omega_0} + \dfrac{\pi}{\omega_1} < t \leqslant T \end{cases}. \quad (3)$$

The variations in function (3) allow changes of vibration amplitudes and frequencies in the vibrational process to be considered. The variation of this function has not been considered in papers [12–15] to describe the vibrational process. The results of the paper [11] have been used to evaluate function (3).

It is assumed, when the crack is open, the influence of the elastic energy of a strain in the affected zone of shell material disappears. It is equivalent to removing material in this area. The variants of removing the material are shown in Figure 2, which has the designations h_1 the depth of the crack, Δ the dimension of zone of crack influence along the shell. The relation between Δ, h, h_1 obtained by the authors in reference [11] has the form: $\Delta = 2{\cdot}5Pl(1-v)h_1/2\pi aGh^2$ where G is shear modulus and P the stretching force.

3. DETERMINATION OF PARAMETERS OF VIBRATIONS AND DIAGNOSTIC FUNCTIONS

Relay's method is applied to calculate the vibration frequency of the shell. The vibration frequencies are determined from the principles of energy conservation of vibrations. $K + P = const$ where K and P are, respectively, kinetic and potential energy of the shell vibration. In the process of vibration, the shell can adopt transient positions where $K = K_{max}$, $P = 0$ and where $K = 0$, $P = P_{max}$. Then, using the conservation principle, $K_{max} = P_{max}$, K_{max} and P_{max} are calculated by the well-known formulae as stated in reference [20].

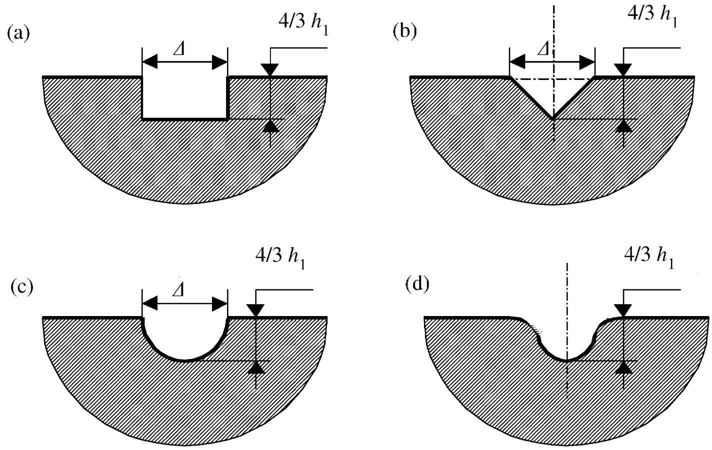

Figure 2. Effect of the crack modelled by (a) by rectangle; (b) triangle; (c) ellipse; (d) by the function $f(x) = h/2 - \frac{4}{3}h_1 \mathrm{e}^{-(x-l/2)^2/8\Delta^2}$.

By decomposing function (3) in a Fourier's series

$$w(x,t) = \frac{a_0}{2} + \sum_{k=1}^{n} (a_k \cos k\bar{\omega}t + b_k \sin k\bar{\omega}t). \tag{4}$$

The coefficients of the decomposition appear as:

$$a_k = \frac{4(-1)^k A_0 \bar{\omega}}{2\pi}\left(\frac{\omega_0}{\omega_0^2 - k^2\bar{\omega}^2} - \frac{\zeta\omega_1}{\omega_1^2 - k^2\bar{\omega}^2}\right)\sin\frac{\pi x}{l}\cos\frac{k\pi\bar{\omega}}{2\omega_1}, \quad k = 0, 1, 2, \cdots,$$

$$b_k = 0, \quad k = 1, 2, \cdots, \tag{5}$$

where $\zeta = A_1/A_0$.

Let one consider the diagnostic functions:

(1) The ratio of amplitudes in different parts of a vibration cycle;

(2) The ratio of frequencies in different parts of a vibration cycle;

(3) The function comparing the ratio of the kth harmonic amplitude appearing in case of a crack to the amplitude of the first harmonic, i.e.,

$$d_k = \sqrt{a_k^2 + b_k^2}/\sqrt{a_1^2 + b_1^2}, \tag{6}$$

Values of these three diagnostic functions are obtained for function (3). The models of the shell vibrations with dispersed and local transverse cracks are constructed. The models of the shell vibrations with dispersed and single transverse cracks can be used for the diagnosis of the degree of fissuring in rocks. The shell modes is shown schematically in Figure 3. The damages to the shell penetrates as far as $\frac{4}{3}h_1$ and coincides with shell length l. The distance between cracks is assumed to be smaller than $\Delta/2$.

The ratio of vibration frequencies in case of the shell with dispersed cracks is obtained by means of Relay's energy method and has the form

$$\frac{\omega_0}{\omega_1} = \left(\frac{h^3\pi^4/12l^4 + h/a^2}{\frac{\pi^4}{l^4}\frac{(3h-8h_1)^3 + 27h^3}{648} + \frac{3h-4h_1}{3a^2} - \frac{\mu\pi^2}{l^2 a}\left(\left(\frac{h}{2} - \frac{4h_1}{3}\right)^2 - \frac{h^2}{4}\right)}\right)^{1/2}. \tag{7}$$

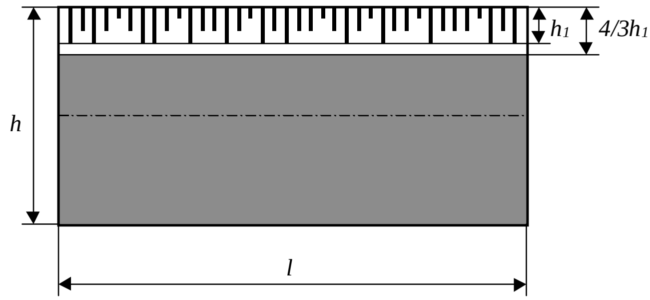

Figure 3. The scheme of a part of the shell surface with dispersed cracks.

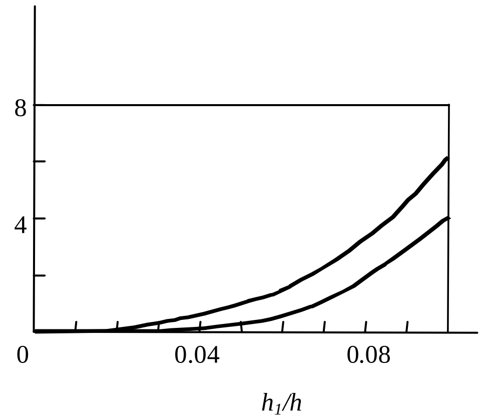

Figure 4. Diagnostic functions for dispersed cracks. The upper curve portrays $(w_1/w_0) \times 10^{-2}$ and the lower curve $(A_1/A_0 - 1) \times 10^{-2}$.

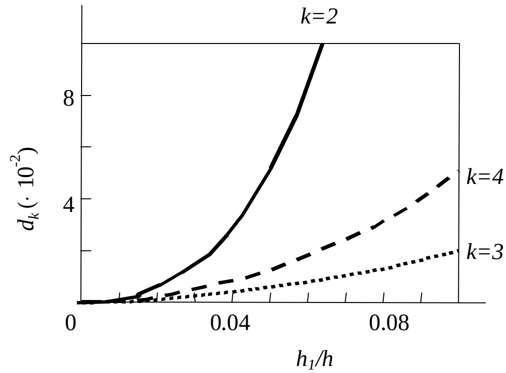

Figure 5. Diagnostic functions for dispersed cracks with $k > 2(-)$; $k = 3$ (- - -); $k = 4$ (– – –).

The ratio of amplitudes has the similar form. In Figures 4 and 5 the graphs of diagnostic functions are shown.

The ratios of vibration frequencies and amplitudes for the single crack whose effect is modelled by the rectangle is obtained by means of Relay's energy method and are

$$\frac{\omega_0}{\omega_1} = \left(\frac{h\pi^2}{2Hl}\right)^{1/2}, \qquad \frac{A_1}{A_0} = \left(\frac{l}{2H}\left(\frac{h^3}{12}\frac{\pi^4}{l^4} + \frac{h}{a^2}\right)\right)^{1/2}, \tag{8}$$

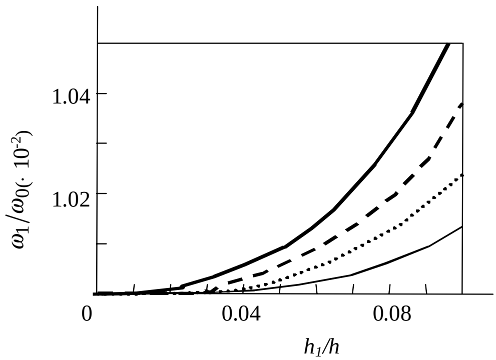

Figure 6. Diagnostic function (w_1/w_0) for local cracks modelled in Figure 2. —, influence of the crack shown in Figure 2(a); – – –, 2(b); · · ·, 2(c); —, 2(d).

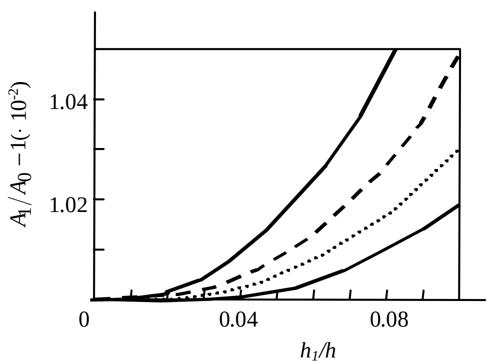

Figure 7. Diagnostic function ($A_1/A_0 - 1$) for local cracks. Key as for Figure 6.

where for brevity,

$$H = \left(\frac{h^3}{12}\frac{\pi^4}{l^4} + \frac{h}{a^2}\right)\left(\frac{l-\Delta}{2} - \frac{l}{2\pi}\sin\frac{\pi\Delta}{l}\right) + \left(\frac{\pi^4}{3l^4}\left(\left(\frac{h}{2} - \frac{4h_1}{3}\right)^3 + \frac{h^3}{8}\right)\right.$$
$$\left. + \frac{h - \frac{4}{3}h_1}{a^2} - \frac{v\pi^2}{l^2 a}\left(\left(\frac{h}{2} - \frac{4h_1}{3}\right)^2 - \frac{h^2}{4}\right)\right)\left(\frac{\Delta}{2} + \frac{l}{2\pi}\sin\frac{\pi\Delta}{l}\right). \tag{9}$$

The formulae for other models of the influence of cracks have cumbersome form and they are derived in this paper. The values of the diagnostic functions for a single crack are illustrated by the graphs (Figures 6–8).

Figures 5 and 8 show that the most informative function amongst those considered is that of d_2. Thus, the appearance of the second harmonic enables one to diagnose the geometrical parameters of the crack.

The experimental approaches to the determination of the values of the diagnostic function (6) for a cantilever bar are offered in paper [16]. Paper [17] outlines the experimental method of finding the vibration parameter, the value of which describes with the relationship of the frequencies.

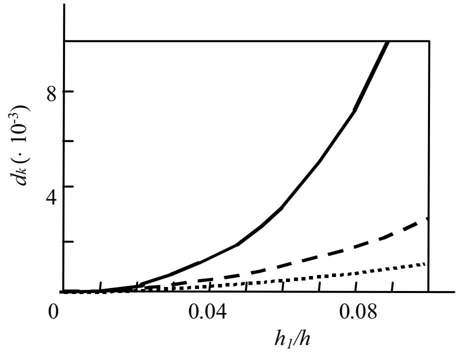

Figure 8. Diagnostic function (d_k) for the single crack with $k = 2$ (—); $k = 4$ (– – –) and $k = 6$ (· · · ·).

4. CONCLUSIONS

The values of bending vibration parameters of circular cylindrical shell have been found: the frequencies and amplitudes of vibration of the shell without cracks; the shell vibration frequencies with a transverse crack, and the frequencies of shell vibration with a closing transverse crack which can be used when constructing the diagnostic functions of crack detection. The graphs of the diagnostic functions have been presented for the ratio of amplitudes in different parts of a vibration cycle; the ratio of frequencies in different parts of a shell vibration cycle; the function describing the ratio of the k^{th} harmonic amplitude to that of the first harmonic in the presence of a crack.

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